Math 647 – Applied PDE
Homework 4 – Hints!

Spring 2019

Here, I give some hints to some selected exercises...

2.3.2 If we set $\Omega(T) = \{0 \leq x \leq L, 0 \leq t \leq T\}$ for each $T > 0$, notice that

$$0 < T_1 < T_2 \quad \Rightarrow \quad \Omega(T_1) \subset \Omega(T_2).$$

What does this mean about $M(T_1)$ and $M(T_2)$?

2.3.4 For (a), use the "strong" maximum principle discussed right after the statement of the "maximum principle" in Strauss.
For (b), show that both the functions $u(x,t)$ and $u(1-x,t)$ satisfy the same PDE and initial / boundar conditions and recall we have uniqueness of solutions.
For (c), by "energy method" Strauss means the method used to give the second proof of uniqueness on pg. 44 of his book. In particular, use that energy is decreasing here...

2.3.6 Find a PDE with boundary / initial conditions satisfied by $u - v$....

2.4.6 Expanding on the hint in the book, notice if we set $I = \int_0^\infty e^{-x^2} dx$, then

$$I^2 = \left(\int_0^\infty e^{-x^2} dx\right) \cdot \left(\int_0^\infty e^{-y^2} dy\right) = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy.$$

Using polar coordinates, evaluate this integral and determine the value of I.

2.4.8 For a given $\delta > 0$, what is $\max_{\delta \leq |x| < \infty} e^{-x^2/4kt}$ as a function of t?